

Homework 1

If X is a topological space, a point $x \in X$ is called an **isolated point** of X if the one point set $\{x\}$ is open in X .

(H1.1) Let (X, T) be a TDS and assume that X is metrizable without isolated points. The following are equivalent:

- (i) (X, T) is forward transitive.
- (ii) For any nonempty open subset U of X , $\bigcup_{n \geq 1} T^n(U)$ is dense in X .
- (iii) For any nonempty open subset U of X , $\bigcup_{n \geq 0} T^n(U)$ is dense in X .
- (iv) For any nonempty open subsets U, V of X , there exists $n \geq 1$ such that $T^n(U) \cap V \neq \emptyset$.
- (v) For any nonempty open subsets U, V of X , there exists $n \geq 0$ such that $T^n(U) \cap V \neq \emptyset$.
- (vi) For any nonempty open subset U of X , $\bigcup_{n \geq 1} T^{-n}(U)$ is dense in X .
- (vii) For any nonempty open subset U of X , $\bigcup_{n \geq 0} T^{-n}(U)$ is dense in X .
- (viii) For any nonempty open subsets U, V of X , there exists $n \geq 1$ such that $T^{-n}(U) \cap V \neq \emptyset$.
- (ix) For any nonempty open subsets U, V of X , there exists $n \geq 0$ such that $T^{-n}(U) \cap V \neq \emptyset$.
- (x) The set of forward transitive points is residual.

(H1.2) Let (X, T) be an invertible TDS and assume that X is metrizable without isolated points. Then (X, T) is forward transitive if and only if (X, T) is transitive.

(H1.3) Let (\mathbb{S}^1, R_a) be the rotation on the circle group (See Example 1.1.4 in the lecture). Prove that (\mathbb{S}^1, R_a) is transitive if and only if a is not a root of unity.

(H1.4) Is it true that the product of two forward transitive TDSs is forward transitive?

Definition . A shift space is **irreducible** if for every ordered pair of blocks $u, v \in \mathcal{B}(X)$, there is a $w \in \mathcal{B}(X)$ such that $uwv \in \mathcal{B}(X)$.

The empty space and the full shift $W^{\mathbb{Z}}$ are trivial examples of irreducible shift spaces.

(H1.5)

- (i) Prove that a shift space is irreducible if and only if it is transitive.
- (ii) Give some nontrivial examples of irreducible shift spaces.