SNSB
Winter Term 2010/2011
Ergodic Ramsey Theory
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Homework 2

(H2.1) Let (X, T) be an invertible TDS. The following are equivalent:

(i) (X, T) is minimal.

(ii) Every $x \in X$ is forward transitive.

(iii) There are no non-trivial closed strongly T-invariant sets in X.

(iv) Every $x \in X$ is transitive.

(v) There are no non-trivial open strongly T-invariant sets in X.

(vi) $X = \bigcup_{n\in\mathbb{Z}} T^n(U)$ for every nonempty open subset U of X.

(vii) For every nonempty open subset U of X, there are $n_1, \ldots, n_k \in \mathbb{Z}$ such that $X = \bigcup_{i=1}^{k} T^{n_i}(U)$.

(H2.2) Give examples of minimal subsystems of the full shift $W^{\mathbb{Z}}$ and conclude that $W^{\mathbb{Z}}$ is not a minimal TDS when $|W| \geq 2$.

(H2.3) Let (X, T) be a TDS. The following are equivalent:

(i) (X, T) is minimal.

(ii) every point of X is forward transitive and almost periodic.

(iii) there exists a forward transitive point $x_0 \in X$ which is also almost periodic.

(H2.4) Let (X, T) be a TDS and $x \in X$. The following are equivalent:

(i) x is recurrent.

(ii) x is infinitely recurrent.

(H2.5) Consider the full shift $W^{\mathbb{Z}}$. The following are equivalent:

(i) $\mathbf{x} \in W^{\mathbb{Z}}$ is recurrent.

(ii) Every nonempty block of $\mathbf{x}$ occurs a second time.

(iii) Every nonempty block of $\mathbf{x}$ occurs infinitely often.

(H2.6) Consider the full shift $W^{\mathbb{Z}}$.

(i) For any point $\mathbf{x} \in W^{\mathbb{Z}}$, the following are equivalent:

  (a) $\mathbf{x}$ is almost periodic.

  (b) $\mathbf{x}$ is recurrent and the gaps between two consecutive occurences of a given block u in $\mathbf{x}$ are bounded (i.e. for a given block u there exists $M \geq 1$ such that if $\mathbf{x}_{[i,j]}$ and $\mathbf{x}_{[n+i,n+j]}$ are two consecutive occurrences of u, then $1 \leq n \leq M$).

(ii) Give an examples, in the 2-shift $\{0,1\}^{\mathbb{Z}}$, for the sake of simplicity, of

  (a) a recurrent point which is not almost periodic, and

  (b) an almost periodic point which is not periodic.

(H2.7) Let (G, L_a) $(a \in G)$ be the left translation on a compact group (see Example 1.1.3 in the lecture).

(i) G is minimal if and only if G is transitive if and only if $\{a^n \mid n \in \mathbb{Z}\}$ is dense in G.

(ii) Each $g \in G$ is almost periodic.

(iii) Give an example of a TDS (G, L_a) which is not minimal.