SNSB
Winter Term 2010/2011
Ergodic Ramsey Theory
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Homework 3

(H3.1) Give an example showing that the Multiple Recurrence Theorem 1.7.0.3 does not hold if the condition of commutativity of the homeomorphisms is omitted.

(H3.2) Let $l \geq 1$ and $T_1, \ldots, T_l : X \to X$ be commuting continuous mappings of a compact metric space (X, d). Then there exists a multiply recurrent point for $T_1, \ldots, T_l$.

(H3.3) Prove that the following statement is in general false: *For any finite partition of* $\mathbb{N}$*, one of the cells contains an infinite arithmetic progression.*

(H3.4) Let us consider the following statement

(*) Let X be a compact metric space, $T : X \to X$ be a homeomorphism, and $(U_i)_{i \in I}$ be an open cover of X. Then there exists an open set U_{i_0} such that for all $k \geq 1$, $U_{i_0} \cap T^{-n}(U_{i_0}) \cap \ldots \cap T^{-(k-1)n}(U_{i_0}) \neq \emptyset$ for infinitely many n.

(i) Prove **(*)** in two ways:

(a) applying Multiple Recurrence Theorem.

(b) using van der Waerden theorem.

(ii) Deduce van der Waerden theorem from **(*)**.

(H3.5) Prove the multidimensional van der Waerden theorem 2.1.4.1.