SNSB
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Ergodic Ramsey Theory
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Seminar 2

(S2.1)

(i) $(X, 1_X)$ is minimal if and only if $|X| = 1$.

(ii) If (X, T) is minimal, then T is surjective.

(iii) A factor of a minimal TDS is also minimal.

(iv) If a product TDS is minimal, then so are each of its components.

(v) If (X_1, T_{X_1}), (X_2, T_{X_2}) are two minimal subsystems of a TDS (X, T), then either $X_1 \cap X_2 = \emptyset$ or $X_1 = X_2$.

(vi) A disjoint union of two TDSs is never a minimal TDS.

(S2.2) Let (X, T) be a TDS and assume that X is metrizable. For any $x \in X$, the following are equivalent:

(i) x is recurrent.

(ii) $\lim\limits_{k\to\infty} T^{n_k}x = x$ for some sequence (n_k) in $\mathbb{Z}_+$.

(iii) $\lim\limits_{k\to\infty} T^{n_k}x = x$ for some sequence (n_k) in $\mathbb{Z}_+$ such that $\lim\limits_{k\to\infty} n_k = \infty$.

(S2.3)

(i) If $\varphi : (X, T) \to (Y, S)$ is a homomorphism of TDSs and $x \in X$ is recurrent (almost periodic) in (X, T), then $\varphi(x)$ is recurrent (almost periodic) in (Y, S).

(ii) If (A, T_A) is a subsystem of (X, T) and $x \in A$, then x is recurrent (almost periodic) in (X, T) if and only if x is recurrent (almost periodic) in (A, T_A).

(S2.4) Let (X, T) be a TDS and $x \in X$. The following are equivalent:

(i) x is almost periodic.

(ii) For any open neighborhood U of x, there exists $N \geq 1$ such that

$$\mathrm{orb}_+(x) \subseteq \bigcup_{k=0}^{N} T^{-k}(U).$$

(iii) $(\overline{\mathrm{orb}_+}(x), T_{\overline{\mathrm{orb}_+(x)}})$ is a minimal subsystem.

Definition . *A TDS (X,T) is said to be* **isometric** *if there exists a metric d on X inducing the topology of X such that T is an isometry with respect to d.*

(S2.5) Give examples of isometric TDSs.

(S2.6) Let (X,T) be an isometric TDS. Then

(i) (X,T) is minimal if and only if it is forward transitive.

(ii) For every $x \in X$, $(\overline{\mathrm{orb}_+}(x), T_{\overline{\mathrm{orb}_+(x)}})$ is a minimal subsystem. Conclude that every point $x \in X$ is contained in a unique minimal subsystem and that (X,T) is a disjoint union of minimal subsystems.

(iii) Every point $x \in X$ is almost periodic.