SNSB
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Ergodic Ramsey Theory
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Seminar 3

(S3.1) Let $x \in X$ and $\mathbf{x} = (x, \ldots, x) \in X^l_\Delta$ (see the notations from Section 1.7). The following are equivalent:

(i) x is multiply recurrent for $T_1, \ldots, T_l$.

(ii) $\mathbf{x}$ is a recurrent point in $(X^l, \tilde{T})$.

(iii) For all $\varepsilon > 0$ there exists $N \geq 1$ such that $d(\mathbf{x}, \tilde{T}^N \mathbf{x}) < \varepsilon$.

(iv) For all $\varepsilon > 0$ there exists $N \geq 1$ such that $d(x, T_i^N x) < \varepsilon$ for all $i = 1, \ldots, l$.

(S3.2) Let X be a compact Hausdorff topological space, $l \geq 1$, and $T_1, \ldots, T_l : X \to X$ be commuting homeomorphisms. Then

(i) X contains a subset X_0 which is minimal with the property that it is nonempty closed and strongly T_i-invariant for all $i = 1, \ldots, l$.

(ii) For every nonempty open subset U of X_0, there are $M \geq 1$ and $n_{ij} \in \mathbb{Z}, i = 1, \ldots, l,\ j = 1, \ldots, M$ such that $X_0 = \bigcup_{j=1}^{M} \left(T_1^{n_{1j}} \circ \ldots \circ T_l^{n_{lj}}\right)(U)$.

(iii) $(X_0)^l_\Delta$ is strongly $\tilde{T}_i$-invariant for all $i = 1, \ldots, l$.