

## Seminar 9

(S9.1) Prove that the Bernoulli shift is strong mixing.

(S9.2) Any syndetic set  $A \subseteq \mathbb{Z}_+$  has positive lower density.

**Definition .** Let  $(X, d)$  be a metric space,  $A \subseteq X$  and  $\varepsilon > 0$ . We say that  $x, y \in A$  are  $\varepsilon$ -**separated** if  $d(x, y) \geq \varepsilon$ .  $A$  is said to be  $\varepsilon$ -**separated** if every two points of  $A$  are  $\varepsilon$ -separated.

(S9.3) Let  $(X, d)$  be a metric space and  $A \subseteq X$  be totally bounded with  $|A| \geq 2$ . Define for all  $\varepsilon > 0$ , the set

$$S_\varepsilon = \{m \geq 2 \mid \text{there exist } m \text{ } \varepsilon\text{-separated points in } A\} \quad (\text{E.3})$$

Prove that if  $S_\varepsilon$  is nonempty, then it is bounded from above.

(S9.4) Let  $f \in AP(X)$ . Prove that for any  $\varepsilon > 0$  the set

$$A_\varepsilon := \{n \geq 1 \mid \|U_T^n f - f\|_2 < \varepsilon\} \quad (\text{E.4})$$

is syndetic.