

Seminar 4

(S4.1) Consider the full shift $W^{\mathbb{Z}}$. The following are equivalent:

- (i) $\mathbf{x} \in W^{\mathbb{Z}}$ is recurrent.
- (ii) Every nonempty block of \mathbf{x} occurs a second time.
- (iii) Every nonempty block of \mathbf{x} occurs infinitely often.

(S4.2) Let $x \in X$ and $\mathbf{x} = (x, \dots, x) \in X_{\Delta}^l$ (see the notations from Section 1.7). The following are equivalent:

- (i) x is multiply recurrent for T_1, \dots, T_l .
- (ii) \mathbf{x} is a recurrent point in (X^l, \tilde{T}) .
- (iii) For all $\varepsilon > 0$ there exists $N \geq 1$ such that $d(\mathbf{x}, \tilde{T}^N \mathbf{x}) < \varepsilon$.
- (iv) For all $\varepsilon > 0$ there exists $N \geq 1$ such that $d(x, T_i^N x) < \varepsilon$ for all $i = 1, \dots, l$.

(S4.3) Let X be a compact Hausdorff topological space, $l \geq 1$, and $T_1, \dots, T_l : X \rightarrow X$ be commuting homeomorphisms. Then

- (i) X contains a subset X_0 which is minimal with the property that it is nonempty closed and strongly T_i -invariant for all $i = 1, \dots, l$.
- (ii) For every nonempty open subset U of X_0 , there are $M \geq 1$ and $n_{ij} \in \mathbb{Z}, i = 1, \dots, l, j = 1, \dots, M$ such that $X_0 = \bigcup_{j=1}^M (T_1^{n_{1j}} \circ \dots \circ T_l^{n_{lj}})(U)$.
- (iii) $(X_0)_{\Delta}^l$ is strongly \tilde{T}_i -invariant for all $i = 1, \dots, l$.