SNSB
Summer Term 2013
Ergodic Theory and Additive Combinatorics
Laurenţiu Leuştean

28.05.2013

Seminar 5

(S5.1) Let us consider the following statements

(vdW1) Let $r \in \mathbb{Z}_+$ and $\mathbb{N} = \bigcup_{i=1}^{r} C_i$. For any $k \geq 1$ there exists $i \in [1, r]$ such that C_i contains an arithmetic progression of length k.

(vdW2) Let $r \in \mathbb{Z}_+$ and $\mathbb{N} = \bigcup_{i=1}^{r} C_i$. There exists $i \in [1, r]$ such that C_i contains arithmetic progression of arbitrary finite length.

(vdW3) Let $r \in \mathbb{Z}_+$ and $\mathbb{N} = \bigcup_{i=1}^{r} C_i$. For any finite set $F \subseteq \mathbb{N}$ there exists $i \in [1, r]$ such that C_i contains affine images of F.

(vdW4) Let $r \in \mathbb{Z}_+$ and $\mathbb{N} = \bigcup_{i=1}^{r} C_i$. There exists $i \in [1, r]$ such that C_i contains affine images of every finite set $F \subseteq \mathbb{N}$.

Let (**vdWi**$*$), $i = 1, 2, 3, 4$ be the statements obtained from (**vdWi**), $i = 1, 2, 3, 4$ by changing $\mathbb{N}$ to $\mathbb{Z}$ in their formulations.

Prove that (**vdWi**), (**vdWi**$*$), $i = 1, 2, 3, 4$ are all equivalent.

(S5.2) Let us consider the following statement

(*) Let (X, T) be a TDS and $(U_i)_{i \in I}$ be an open cover of X. Then there exists an open set U_{i_0} in this cover such that $U_{i_0} \cap T^{-n}(U_{i_0}) \neq \emptyset$ for infinitely many n.

(i) Prove **(*)** in two ways:

(a) applying Birkhoff Recurrence Theorem.

(b) using the Infinite Pigeonhole Principle (IPP): Whenever $\mathbb{N}$ is coloured into finitely many colours, one of the colour classes is infinite.

(ii) Deduce IPP from **(*)**.