FMI, CS, Master I
Techniques of Combinatorial Optimization
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Seminar 1

(S1.1) Any polyhedron is a convex set.

(S1.2) Prove that

(i) Affine sets are polyhedra.

(ii) Singletons are polyhedra of dimension 0.

(iii) Lines are polyhedra of dimension 1.

(iv) The unit cube $C_3 = \{x \in \mathbb{R}^3 \mid 0 \leq x_i \leq 1 \text{ for all } i = 1, 2, 3\}$ in $\mathbb{R}^3$ is a full-dimensional polyhedron.

(S1.3) [Farkas lemma - variant] The system $Ax = b$ has a solution $x \geq \mathbf{0}$ if and only if $y^T b \geq 0$ for each $y \in \mathbb{R}^m$ with $y^T A \geq \mathbf{0}^T$.

(S1.4) [Farkas lemma - variant] The system $Ax \leq b$ has a solution if and only if $y^T b \geq 0$ for each $y \geq \mathbf{0}$ with $y^T A = \mathbf{0}^T$.