FMI, CS, Master I
Techniques of Combinatorial Optimization
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Seminar 2

(S2.1) Let $A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$. Then

$$\max\{c^T x \mid x \geq \mathbf{0}, Ax \leq b\} = \min\{b^T y \mid y \geq \mathbf{0}, y^T A \geq c^T\}.$$

(assuming both sets are nonempty).

(S2.2) Let $A \in \mathbb{R}^{m\times n}$ be a TU matrix, let $b \in \mathbb{Z}^m$ and $c \in \mathbb{Z}^n$. Assume that the primal LP $\max\{c^T x \mid Ax \leq b\}$ and dual LP $\min\{b^T y \mid y \geq \mathbf{0}, y^T A = c^T\}$ are bounded. Then they have integer optimal solutions.

(S2.3) Let $A \in \mathbb{R}^{m\times n}$ be a TU matrix, let b, b', d, d' be vectors in $(\mathbb{Z} \cup \{-\infty, +\infty\})^m$ with $b \leq b'$ and $d \leq d'$. Then

$$P = \{x \in \mathbb{R}^n \mid b \leq Ax \leq b', d \leq x \leq d'\}$$

is an integer polyhedron.