FMI, CS, Master I
Techniques of Combinatorial Optimization
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Seminar 3

(S3.1) Let G be the complete graph K_3 on three vertices. Prove that its incidence matrix is not totally unimodular.

(S3.2) Let A be the incidence matrix of a cycle of length 5. Prove that A is a square matrix and compute its determinant.

(S3.3) Verify if the following matrices are totally unimodular:

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

(S3.4) Let G be a graph. Prove that

$$\max\{|M| \mid M \text{ is a matching of } G\} \leq \min\{|S| \mid S \text{ is a vertex cover of } G\}.$$

Show that the complete graph K_3 is an example of a graph where strict inequality holds.

(S3.5) Let $G = (V, E)$ be a bipartite graph and $w : E \to \mathbb{N}$ be a weight function. The maximum weight of a matching in G is equal to the minimum value of $\sum_{v \in V} y_v$, where y ranges over all functions $y : V \to \mathbb{N}$ such that $y_u + y_v \geq w(e)$ for each edge $e = uv$ of G.