FMI, CS, Master I
Techniques of Combinatorial Optimization
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Seminar 4

(S4.1) Figure 1 represents a flow network $N = (D, c, s, t)$.

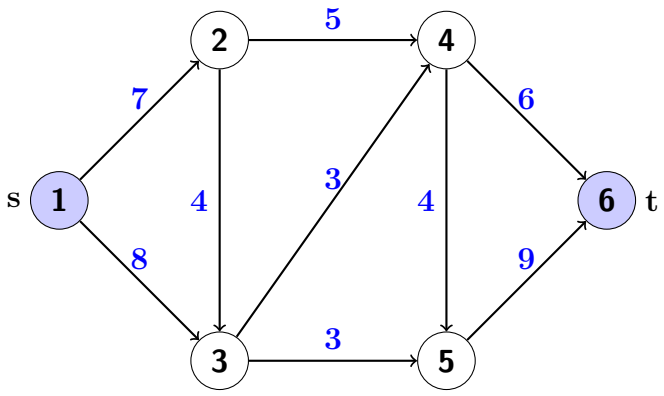

Figure 1: The flow network N

Write the corresponding digraph D and the capacity function c.

(S4.2) Find vectors b, d and a matrix B such that

$$\max\{\text{value}(f) \mid f \text{ is an } s-t \text{ flow for } N\} = \max\{d^T f \mid Bf \leq b\}.$$

(S4.3) Figure 2 represents an s-t flow f for the network N.

(i) Verify that f is an s-t flow. What is the value of f?

(ii) Show that the set $\{(2,4),(3,4),(3,5)\}$ is an s-t cut and compute its capacity.

(iii) Prove that f is a maximum flow.

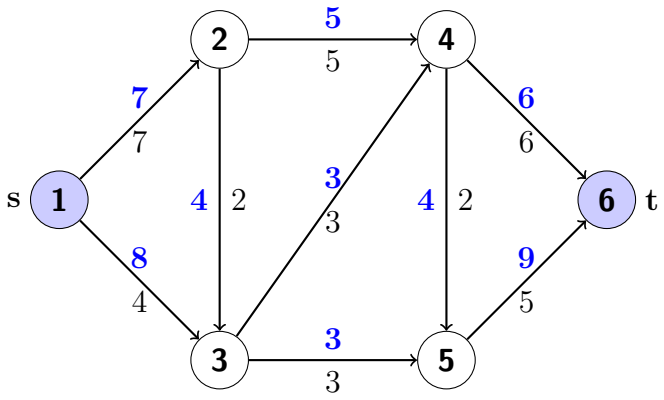

Figure 2: The flow network N with the flow f

(S4.4) Let $N = (D, c, s, t)$ be a flow network and $f : A \to \mathbb{R}$ be an s-t flow. Prove that the value of f is equal to the net amount of flow entering t, that is prove that

$$\text{value}(f) = f(\delta^{in}(t)) - f(\delta^{out}(t).$$

(S4.5) Let $N = (D, c, s, t)$ be a flow network with the property that all capacities are even (that is, $c(a)$ is even for every arc a of D). Prove that the maximum value of a flow is even.