

## Seminar 5

(S5.1) Figure 1 represents a flow network  $N = (D, c, s, t)$ .

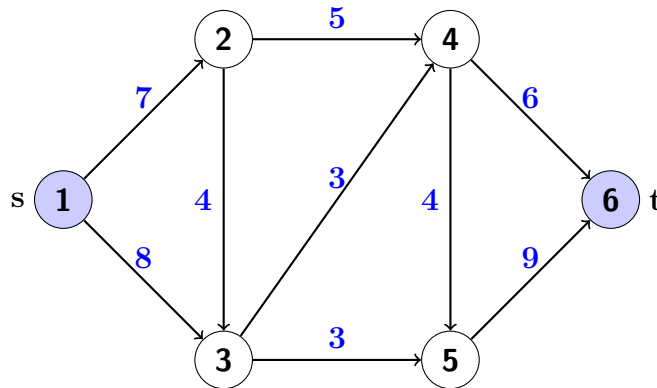


Figure 1: The flow network  $N$

Give two iterations of the Ford-Fulkerson algorithm for  $N$ , considering the path  $P = 1246$  for the first augmentation and  $Q = 1356$  for the second augmentation.

(S5.2) Figure 2 represents a flow network  $N$  and an  $s$ - $t$  flow  $f$  for  $N$ .

- (i) Represent the residual graph  $D_f$  and the residual capacities  $c_f$ .
- (ii) Choose an  $f$ -augmenting path  $P$  of minimum length and compute the flow  $g := f_P^\gamma$ , where  $\gamma = \min_{e \in A(P)} c_f(e)$ .
- (iii) Represent the residual graph  $D_g$  and the residual capacities  $c_g$ . Can you find an  $s$ - $t$  path in  $D_g$ ?
- (iv) What is the maximum value of an  $s$ - $t$  flow for  $N$ ?

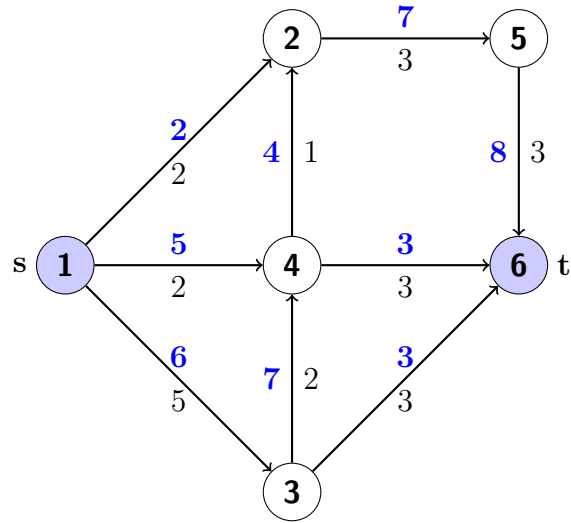


Figure 2: The flow network  $N$  with the flow  $f$

(v) Give an example of an  $s$ - $t$  cut in  $N$  of minimum capacity.

**(S5.3)** Prove Proposition 3.4.2..

**(S5.4)** For any  $s$ - $t$  path  $P$  in  $D$ , prove that  $\chi^P$  satisfies the flow conservation law at every  $v \neq s, t$  and that  $\text{value}(\chi^P) = 1$ .