

## Seminar 6

(S6.1) Give an example where feasible circulations do not exist.

(S6.2) Let  $N = (D, s, t)$  be a unit capacity network,  $k \geq 1$  and  $P_1, \dots, P_k$  be  $k$  arc-disjoint  $s$ - $t$  paths in  $D$ . Then for all  $k \geq 1$ ,

$$f := \chi^{P_1} + \dots + \chi^{P_k}$$

is an  $s$ - $t$   $\{0, 1\}$ -flow  $f$  with  $\text{value}(f) = k$ .

(S6.3) Let  $D = (V, A)$  be a digraph. Prove that

- (i) Each  $s$ - $t$  cut is an  $s$ - $t$  disconnecting arc set.
- (ii) Each  $s$ - $t$  disconnecting arc set of minimum size is an  $s$ - $t$  cut.
- (iii) The minimum size of an  $s$ - $t$  disconnecting arc set coincides with the minimum size of an  $s$ - $t$  cut.

(S6.4) Prove that the incidence matrix  $M$  of a directed graph  $D = (V, A)$  is totally unimodular.