**FMI, Computer Science, Master**
**Advanced Logic for Computer Science**

# Seminar 3

**(S3.1)** Let $\mathcal{L}$ be a first-order language and $\varphi$ be a sentence of $\mathcal{L}$ with the property that for all $m \in \mathbb{N}$,

there exists a finite $\mathcal{L}$-structure $\mathcal{A}$ of cardinality $\geq m$ such that $\mathcal{A} \vDash \neg\varphi$.

Prove that $\neg\varphi$ has an infinite model.

**(S3.2)** Let $\mathcal{L}_{Graf}$ be the language of graphs. Decide if the following affirmations are true or false:

(i) the class of graphs is axiomatizable;

(ii) the class of graphs is finitely axiomatizable;

(iii) the class of finite graphs is axiomatizable;

(iv) the class of finite graphs is finitely axiomatizable;

(v) the class of infinite graphs is axiomatizable;

(vi) the class of infinite graphs is finitely axiomatizable.

**(S3.3)** Let $\mathcal{L}$ be a first-order language, $\mathcal{K}$ be a class of $\mathcal{L}$-structures and $\mathcal{K}^c$ its complement in the class of all $\mathcal{L}$-structures. Prove that if both $\mathcal{K}$ and $\mathcal{K}^c$ are axiomatizable, then both of them are finitely axiomatizabile.

**(S3.4)** Let $\mathcal{L}$ be a first-order language and $\Sigma$ be a set of sentences satisfying

(*) for all $m \in \mathbb{N}$, $\Sigma$ has a finite model of cardinality $\geq m$.

Prove that the class of finite models of $\Sigma$ is not axiomatizable.