FMI, Computer Science, Master
Advanced Logic for Computer Science

Seminar 4

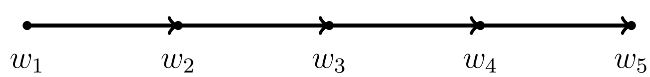
(S4.1) Let $\mathcal{L}$ be a first-order language that contains two unary relation symbols S, T and one binary relation symbol R. Find Skolem normal forms for the following formulas of $\mathcal{L}$:

$$\begin{aligned} \chi &:= \exists y \forall x \exists v ((S(y) \vee R(x,v)) \to (T(v) \to S(y))) \\ \delta &:= \forall x \exists u \forall y \exists v \left((S(u) \to R(v,y)) \vee (S(v) \to T(x))\right). \end{aligned}$$

(S4.2) Let $\mathcal{M} = (W, R, V)$ be a model for ML_0 and w a state in $\mathcal{M}$. Prove that for every formula φ,

$$\mathcal{M}, w \Vdash \Box\varphi \quad \text{iff} \quad \text{for every } v \in W, Rwv \text{ implies } \mathcal{M}, v \Vdash \varphi.$$

(S4.3) Consider the frame $\mathcal{F} = (W = \{w_1, w_2, w_3, w_4, w_5\}, R)$, where Rw_iw_j iff $j = i + 1$:



Let us choose a valuation V such that $V(p) = \{w_2, w_3\}$, $V(q) = \{w_1, w_2, w_3, w_4, w_5\}$ and $V(r) = \emptyset$. Consider the model $\mathcal{M} = (\mathcal{F}, V)$. Prove the following:

(i) $\mathcal{M}, w_1 \Vdash \Diamond\Box p$;

(ii) $\mathcal{M}, w_1 \nVdash \Diamond\Box p \to p$;

(iii) $\mathcal{M}, w_2 \Vdash \Diamond(p \wedge \neg r)$;

(iv) $\mathcal{M}, w_1 \Vdash q \wedge \Diamond(q \wedge \Diamond(q \wedge \Diamond(q \wedge \Diamond q)))$;

(v) $\mathcal{M} \Vdash \Box q$.

(S4.4) Verify if the following formulas of ML_0 are satisfiable:

(i) $\Diamond p \wedge \Box\neg p$;

(ii) $\Diamond p \wedge \Diamond\neg p$.