FMI, Computer Science, Master
Advanced Logic for Computer Science

Seminar 5

(S5.1) Let $\mathcal{M} = (W, R, V)$ be a model for ML_0 and w a state in $\mathcal{M}$. Prove that for every formula φ and any $n \geq 1$,

$$\begin{array}{llcl} (1) & \mathcal{M}, w \Vdash \Diamond^n \varphi & \Longleftrightarrow & \text{there exists } v \in W \text{ such that } R^n wv \text{ and } \mathcal{M}, v \Vdash \varphi \\ (2) & \mathcal{M}, w \Vdash \Box^n \varphi & \Longleftrightarrow & \text{for every } v \in W, R^n wv \text{ implies } \mathcal{M}, v \Vdash \varphi. \end{array}$$

(S5.2) Prove that for every $p, q \in PROP$ the formula

$$\Box(p \to q) \to (\Box p \to \Box q)$$

is valid in the class of all frames for ML_0.

(S5.3) Prove that for any formula φ,

$$\Diamond\varphi \leftrightarrow \neg\Box\neg\varphi$$

is valid in the class of all frames for ML_0.

(S5.4) Let $p \in PROP$. Prove that the formula

$$\Box p \to \Diamond p$$

is not valid in the class of all frames for ML_0.

(S5.5) Let $p, q \in PROP$. Verify if the following formulas are valid in the class of all frames for ML_0:

(i) $p \to \Diamond p$.

(ii) $\Box p \wedge \Diamond q \to \Diamond(p \wedge q)$.