FMI, Computer Science, Master
Advanced Logic for Computer Science

Seminar 6

(S6.1) Let Λ be a normal logic. Prove that for any formulas φ, ψ,

(i) $\vdash_\Lambda \varphi \to \psi$ implies $\vdash_\Lambda \Box\varphi \to \Box\psi$.

(ii) $\vdash_\Lambda \varphi \leftrightarrow \psi$ implies $\vdash_\Lambda \Box\varphi \leftrightarrow \Box\psi$.

(S6.2) Prove that for any formulas φ, ψ,

$$\vdash_{\boldsymbol{K}} \Box\varphi \vee \Box\psi \to \Box(\varphi \vee \psi).$$

(S6.3) Let $\Gamma \cup \{\varphi, \psi\}$ be a set of formulas. Prove that

$$\text{if } \Gamma \vdash_\Lambda \varphi \text{ and } \Gamma \vdash_\Lambda \varphi \to \psi, \text{ then } \Gamma \vdash_\Lambda \psi.$$

(S6.4) Let Λ be a normal logic. Prove that for any formulas φ, ψ,

(i) $\vdash_\Lambda \varphi \to \psi$ implies $\vdash_\Lambda \Diamond\varphi \to \Diamond\psi$.

(ii) $\vdash_\Lambda \varphi \leftrightarrow \psi$ implies $\vdash_\Lambda \Diamond\varphi \leftrightarrow \Diamond\psi$.

(S6.5) Prove that for any formulas φ, ψ,

(i) $\vdash_{\boldsymbol{K}} \varphi \to \psi$ implies $\vdash_{\boldsymbol{K}} \Box\Diamond\varphi \to \Box\Diamond\psi$.

(ii) $\vdash_{\boldsymbol{K}} \Diamond\varphi \vee \Diamond\psi \to \Diamond(\varphi \vee \psi)$.