FMI, Computer Science, Master
Logic for Multiagent Systems

Exam

(P1) [1.5 points] Let $\mathcal{L}$ be a first-order language that contains

- two unary relation symbols S, T;
- a unary function symbol f;
- a constant symbol c.

Find prenex normal forms for the following formulas of $\mathcal{L}$:

$$\begin{aligned} \varphi &:= \forall x S(x) \wedge \neg \exists y S(y), \\ \psi &:= \neg \forall y \left(f(y) = c \to \exists x S(x) \right) \to \left(\exists x T(x) \vee \forall y T(y) \right). \end{aligned}$$

(P2) [2 points] Let $p, q \in PROP$. Verify if the following formulas are valid in the class of all frames for ML_0:

(i) $\Box(p \wedge q) \to (\Box p \wedge \Box q)$;

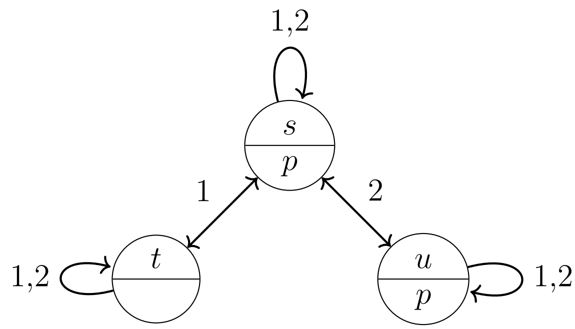
(ii) $\Box p \to p$.

(P3) [1.5 points] Prove the following for any formulas φ, ψ of ML_0:

(i) $\vdash_{\boldsymbol{K}} \Box\varphi \to (\Box\psi \to \Box\varphi)$;

(ii) $\vdash_{\boldsymbol{K}} \psi \to \varphi$ implies $\vdash_{\boldsymbol{K}} \Diamond\Box\psi \to \Diamond\Box\varphi$.

(P4) [1.5 points] Consider the model $\mathcal{M} = (W, \mathcal{K}_1, \mathcal{K}_2, V)$ for epistemic logic represented as follows:



Verify if the following are true:

(i) $\mathcal{M}, t \Vdash K_1 p$;

(ii) $\mathcal{M}, t \Vdash \neg K_2 \neg K_1 p$.

(P5) [1.5 points] Let $\mathcal{M}_c$ be the model describing the simple card game. Prove the following:

(i) $\mathcal{M}_c, (A, B) \models 1A \wedge 2B$;

(ii) $\mathcal{M}_c, (A, B) \models K_1 \neg K_2 1A$.

(P6) [1 point] Let $\mathcal{M}$ be the model for the muddy children puzzle with $n = 3$. Prove the following:

(i) $\mathcal{M}, (1, 0, 1) \Vdash K_1 \neg p_2$;

(ii) $\mathcal{M}, (1, 0, 1) \Vdash K_2 p_3$.